

Akira Tsuchida  
Emiko Takyo  
Kazunori Taguchi  
Tsuneo Okubo

## Kinetic analyses of colloidal crystallization in shear flow

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A. Tsuchida · E. Takyo · K. Taguchi  
T. Okubo (✉)  
Department of Applied Chemistry and  
Graduate School of Materials Science,  
Gifu University, Gifu 501-1193, Japan  
E-mail: okubotsu@apchem.gifu-u.ac.jp  
Fax: +81-58-2932628

**Abstract** Colloidal crystallization kinetics is studied in the shear flow of a suspension of colloidal silica spheres (110 nm in diameter), using a continuously-circulating type of stopped flow cell system. The crystallization rate from a suspension containing a small amount of nuclei and/or single crystals is high compared with that from a suspension containing no nuclei and/or single crystals. Crystal growth takes place

at shear rates smaller than  $3.4 \text{ s}^{-1}$  and at sphere concentrations higher than a volume fraction of 0.004.

**Keywords** Colloidal crystals · Crystallization kinetics · Crystal growth rate · Shear rate · Reflection spectra

### Introduction

The kinetics of colloidal crystallization has been studied by several researchers up to now for two main types of crystal suspensions: diluted and deionized aqueous suspensions where the colloidal particles are covered with electric double layers (called soft-sphere systems) [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21], and concentrated suspensions in refractive index-matched organic solvents (called hard-sphere systems) [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40]. This classification into soft- and hard-sphere systems depends largely on the ionic concentration of the suspension; in other words we get “soft” crystals in a completely deionized environment, and “hard” crystals in the presence of large numbers of ions – a rather large amount of sodium chloride, for example.

Many researchers, including us, clarified that colloidal crystals are formed by Brownian movement of colloidal particles in a closed vessel, and also by the interparticle repulsion leading to minimal dead space [8, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56]. In other words, particles form crystal-like

arrangements owing to the Brownian movement of the particles that leads to maximum packing density.

We have studied both nucleation and crystallization processes in exhaustively deionized and highly-diluted aqueous suspensions via dynamic light scattering measurements, from the half-widths in the sharpened reflection peak, and from increases in the reflection peak intensity [8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. Though the crystallization process was surprisingly fast, reflection spectroscopy could follow crystallization rates of the order of milliseconds. Nucleation rates increased drastically as sphere concentration increased. Crystal growth rates also increased substantially as the sphere concentration increased [11, 12], but only in a narrow concentration range. Furthermore, nucleation and crystallization rates were very sensitive to the degree of deionization of the aqueous suspensions. Recently, we further studied the kinetics of colloidal crystallization in a sinusoidal electric field [14], in a wide range of sphere concentrations [15], in the presence of salt [16], in alcoholic organic solvents [13], and in organic solvents such as acetonitrile for silica spheres coated with polymers [20, 21]. Microgravity experiments exploring the kinetics

of colloidal crystallization, performed on an aircraft, have also been made by our group [17, 18, 19]. In the course of these studies, we recognized that the colloidal crystallization rate is also strongly dependent on the shear rate of the suspension.

In this work, colloidal crystallization rates were measured using a continuously-circulating type of stopped flow cell system. The colloidal crystallization rates were measured as a function of shear rate of the circulating suspension before and during crystallization in the continuous flow state. These experiments are called initial and final shear flow analyses, respectively, in this work.

## Experimental

### Materials

Monodispersed colloidal silica spheres (CS91) were kindly donated by Catalyst & Chemicals Ind. Co. (Tokyo). The diameter ( $d$ ), standard deviation ( $\delta$ ) from the mean diameter, polydispersity index ( $\delta/d$ ), and specific gravity were 110 nm, 4.5 nm, 0.041, and 2.2, respectively. The values of  $d$  and  $\delta$  were determined by electron microscope (JEM-2000FX, JEOL, Tokyo). The charge density of the spheres was determined to be  $0.48 \mu\text{C}/\text{cm}^2$  for strongly acidic groups by conductometric titration with a Horiba model DS-14 conductivity meter. The sphere sample was purified carefully several times using an ultrafiltration cell (model 202, membrane: Diaflo XM, Amicon, Lexington, KY), and then treated with a mixed bed of cation- and anion-exchange resins (Bio-Rad, AG501-X8 (D), 20–50 mesh) for more than five years. The sample suspensions (volume fractions:  $\phi=0.002, 0.003, 0.004, 0.005$  and  $0.006$ ) were in most cases deionized further for 30 min after preparation by circulation in the cell system at the shear rate of  $15 \text{ s}^{-1}$ . The shear rates for the initial and final analyses were changed from 0 to  $34 \text{ s}^{-1}$ . The water used for sample suspension was obtained from a Milli-Q water system (Milli-RO Plus and Milli-Q Plus, Millipore, Bedford, Mass). Experiments were made at  $25 \pm 1.5^\circ\text{C}$  in an air-conditioned room.

### Kinetics experiments for colloidal crystallization

The observation cell system, which was the same as one reported previously for the microgravity experiments [17, 18, 19], consists of a quartz observation cell ( $40 \times 10 \times 2 \text{ mm}$ ), a column of cation- and anion-exchange resins (Bio-Rad), and a peristaltic pump (Masterflex 7524-10, IL). The pump circulates the colloidal suspension first to the resin column and then to the observation cell in order to deionize the suspension continuously. It has been already checked, using reflection spectroscopy, that  $15 \text{ s}^{-1}$  of shear rate almost completely breaks the crystals in the system. A light beam from a halogen lamp (Hayashi LA-150SX, Tokyo) hits the cell wall through a Y-type optical fiber cable, and the reflection spectra are taken on a photonic multichannel analyzer (PMA-50, Hamamatsu Photonics, Hamamatsu).

When the colloidal suspension is passed into the narrow observation cell, the crystals are melted away by the flow shear. In this observation cell, a flow rate of  $1.0 \text{ ml/s}$  corresponds to a shear rate of  $1.7 \text{ s}^{-1}$ . After stopping the flow the crystallization starts. The crystal growing process was followed from the growth of the Bragg reflection peaks. In our experimental conditions the suspension in the flow cell (2 mm in path length) was transparent.

Therefore, the reflection spectroscopy in our experiments will afford information on the crystal growing processes for the whole depth of the suspension in the cell.

Two kinetic analyses were made in this work. The first analysis was made for the measurements of the dependence of the colloidal crystallization rate on the initial shear rate ( $q_i$ ). In these measurements, the initial equilibrium state was attained at various initial shear rates ( $q_i = 1.7\text{--}34 \text{ s}^{-1}$ ) in the measurement cell. Then the flow was stopped suddenly and the crystallization coefficient was measured by reflection spectroscopy. The second analysis was performed for measurements of the dependence of the colloidal crystallization rate on the final shear rate ( $q_f$ ). In these measurements, the initial equilibrium state was fixed at the shear rate of  $34 \text{ s}^{-1}$ . This shear rate is quite enough to break all of the crystals in the cell. Then, the shear rate decreased to various final continuous shear rates ( $q_f = 1.7\text{--}15 \text{ s}^{-1}$ ) and the crystallization rate coefficient was measured.

## Results and discussion

### Effective soft-sphere model for colloidal crystallization

According to the effective soft-sphere model, which is a simple but very convenient assumption for the colloidal crystallization, crystal-like ordering of the single component spheres occurs when the effective diameter ( $d_{\text{eff}}$ ) of the spheres containing the Debye-screening length ( $D_1$ ) is close to or larger than the intersphere distance ( $l$ ):

$$d_{\text{eff}} = (d + 2 \times D_1) > l, \quad (1)$$

$$D_1 = \left( \frac{4\pi e^2 n}{\epsilon k_B T} \right)^{-1/2}, \quad (2)$$

where  $d$  is the sphere diameter,  $e$  is the electronic charge,  $\epsilon$  is the dielectric constant of the solvent,  $k_B$  is the Boltzmann constant, and  $n$  is the concentration of free-state cations and anions in suspension which is given by  $n = n_c + n_s + n_o$ , where  $n_c$  is the concentration (number of ions per  $\text{cm}^3$ ) of diffusible counterions,  $n_s$  is the concentration of foreign salt, and  $n_o$  is the concentration of both  $\text{H}^+$  and  $\text{OH}^-$  from the dissociation of water. At  $\phi = 0.005$  for the CS91 spheres, for example,  $d_{\text{eff}}$  was calculated to be 800 nm. Here, the fraction of free-state counterions ( $\beta$ ) was assumed to be 0.1. This value of  $d_{\text{eff}}$  was larger than the intersphere distance ( $l$ ), 580 nm, calculated assuming the sphere distribution of a simple cubic lattice. That  $d_{\text{eff}} > l$  lends strong support to the soft-sphere model.

### Kinetic analyses of colloidal crystallization

The induction period (the period before the onset of crystal growth) was too short to be observed for most kinetic measurements in this work, considering the comparatively high sphere concentrations [14]. Therefore, the

classical nucleation rates were also not obtained for this study. For reflection spectroscopy, the size of the colloidal single crystals from homogeneous nucleation,  $L$ , is estimated from the intensity of the reflection peak,  $I$  [26]:

$$I \propto N_{\text{cryst}} L^3 \propto L^3, \quad (3)$$

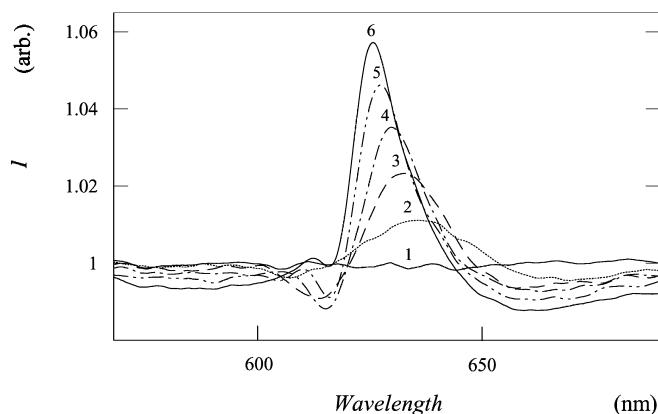
where  $N_{\text{cryst}}$  is the number of single crystals in the reflecting volume, which is directly proportional to the number concentration of crystals in the final stage of the crystallization process (equal to the total number of nuclei formed in the whole course of crystallization). Therefore, the rates of crystallization,  $\nu$ , should be evaluated from the slopes of the plots of the cube root of each peak intensity with time [17]. However, preliminary analyses of these plots gave large uncertainties because the concentrations of the suspension ranged from  $\phi = 0.001$  to 0.006 (rather high). We therefore introduced the rate coefficient,  $k$ , into this work instead of  $\nu$ . The values of  $k$  were obtained as the reciprocal period where the initial linear line in the peak intensities intersects two horizontal lines, giving initial and final times at  $t = t_i$  and  $t_f$ , respectively; in other words:

$$k = \frac{1}{t_f - t_i}. \quad (4)$$

The values of  $t_i$ , which correspond to crystallization induction times, were very close to zero in this work, as mentioned above.

#### Crystallization rates after stopping the flow under shear flow

Typical reflection spectra taken during the crystallization of CS91 spheres at  $\phi = 0.005$  are shown in Fig. 1. The colloidal suspension was initially circulated at a shear rate of  $q_i = 34 \text{ s}^{-1}$  for more than 3 min, and the quasi-

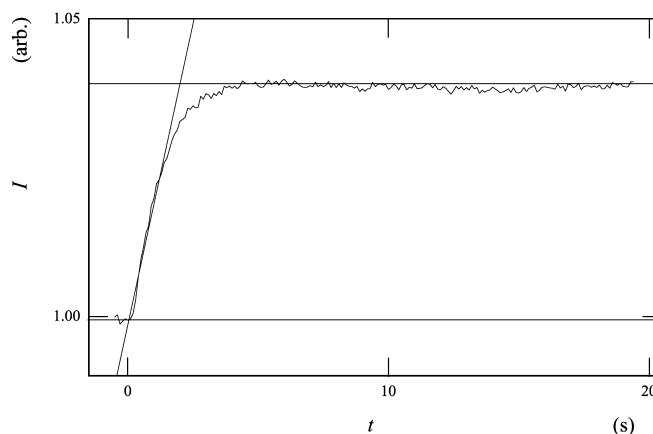


**Fig. 1** Reflection spectra for colloidal crystals of CS91 spheres at 25 °C.  $\phi = 0.005$ ;  $q_i = 34 \text{ s}^{-1}$ ;  $q_f = 0 \text{ s}^{-1}$ . Curve 1: 0 s; 2: 0.7 s; 3: 1.3 s; 4: 2.1 s; 5: 4.4 s; 6: 19.4 s after stopping the flow

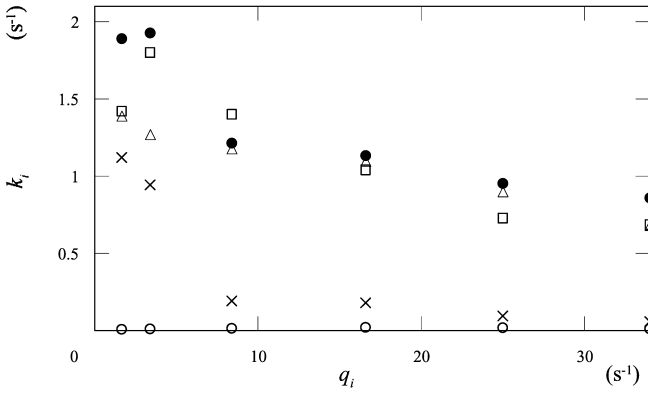
equilibrium state was attained between the crystallization and melting of the sample in the observation cell. The suspension was then stopped suddenly at  $t = 0$ , and further crystal growth processes started. Spectra were obtained from  $t = 0$  to 19.4 s after stopping the pump. A short time after stopping the flow, we noted the appearance of a broad peak in the higher wavelength region at around 635 nm. Over time, the peak intensity increased with a sharpened peak profile, and the peak wavelengths decreased to around 625 nm. This shift in the peak wavelength is explained by the fact that the metastable and expanded crystal structures are formed first and then the crystals become more stable and dense [16, 17, 18, 19]. The peak wavelength observed is reasonable because the secondary Bragg peak wavelength calculated from the sphere concentration is 633 nm for the simple cubic lattice. The Bragg peaks in Fig. 1 are assigned to face-centered cubic (fcc) lattices because another peak from body-centered cubic (bcc) lattices appeared at lower wavelengths for suspensions of much lower concentration [17]. In the present work, all of the kinetic analyses were made for reflection peaks from fcc lattices.

A typical example of the time dependence of the peak intensities is shown in Fig. 2. As is clear from the figure, crystal growth was almost complete by  $\sim 5$  s after stopping the flow.

Figure 3 shows the rate coefficient  $k_i$  for the colloidal crystallization, plotted against  $q_i$  for the sphere concentration from 0.002 to 0.006. Here, the subscript “i” indicates the initial state suspension. The values of  $k_i$  are clearly dependent on  $\phi$  and  $q_i$ . In the concentration range from  $\phi = 0.003$  to 0.006,  $k_i$  decreases as  $q_i$  increases. The dependency of  $k_i$  on  $q_i$  is correlated directly to the rigidity of the colloidal crystal formed in the final state. Furthermore, the partial destruction of the nuclei and/or crystals occurs in the shear flow. Reflection spectroscopy detected the flowing crystals when the suspension was circulated with small  $q_i$ . At small  $q_i$ ,



**Fig. 2** Reflection peak intensities during the colloidal crystallization of CS91 spheres at 25 °C.  $\phi = 0.005$ ;  $q_i = 25 \text{ s}^{-1}$ ;  $q_f = 0 \text{ s}^{-1}$



**Fig. 3** Rate coefficients ( $k_i$ ) as a function of initial shear rate ( $q_i$ ); CS91 spheres at 25 °C;  $q_f=0 \text{ s}^{-1}$ ; unfilled circles:  $\phi=0.002$ ;  $\times$ :  $\phi=0.003$ ; triangles:  $\phi=0.004$ ; squares:  $\phi=0.005$ ; filled circles:  $\phi=0.006$

especially for  $q_i \leq 3.4 \text{ s}^{-1}$ , the crystals and/or nuclei in the suspension will not break up thoroughly, and it is highly plausible that the crystallization is enhanced in the flowing suspension. This means that the crystal growth rate from pre-existing crystals and/or nuclei is higher than that from completely liquid suspension. It should be mentioned that the nucleation time is short enough (less than 20 ms from the resolution of the time-resolved reflection spectra) in the present experimental condition as described above. At  $\phi=0.002$ ,  $k_i$  was independent of  $q_i$ , although the values were small (of the order of  $10^{-2} \text{ s}^{-1}$ ). At low sphere concentrations neither crystals nor nuclei must form in the shear flow, because the rigidity of the crystals is very small.

According to classical crystal growth theory [17, 57, 58], the crystal growth rate  $v$  is given by:

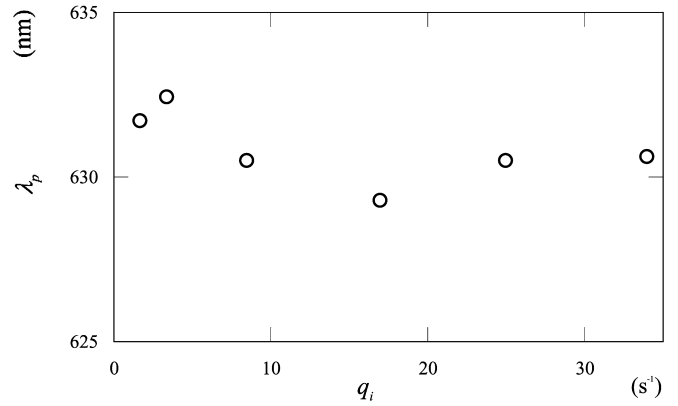
$$v = v_\infty - v_\infty \frac{\phi_c}{\phi}, \quad (5)$$

where  $v_\infty$ ,  $\phi_c$  and  $\phi$  are the maximum crystallization rate, the critical sphere concentration of melting (in volume fraction), and the sphere concentration, respectively. Therefore,  $v$  is expected to increase proportionally with increasing  $1/\phi$ . This linear relationship between  $v$  and  $1/\phi$  is obtained when rather large experimental errors in the  $k_i$  values are taken into account.

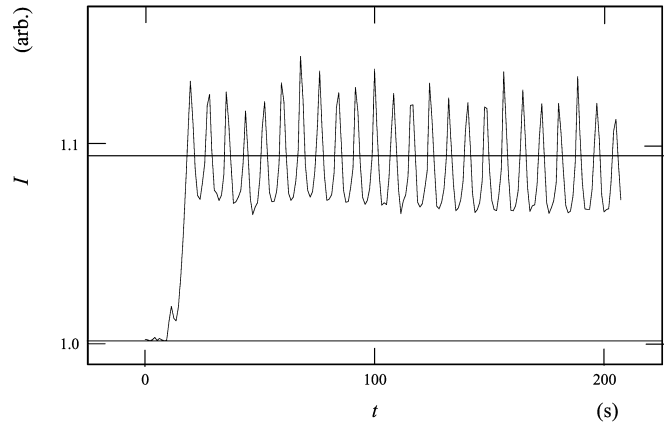
Figure 4 shows the reflection peak wavelength observed at  $t=19.4 \text{ s}$  after stopping the flow, as a function of  $q_i$ . Clearly,  $\lambda_p$  was independent of  $q_i$  to within an accuracy of 0.5%. This means that the crystals formed in the final step are the same irrespective of  $q_i$ .

Crystallization rates when the final stage of the suspension during crystallization is in shear flow

Figure 5 shows a typical example of the reflection peak intensities measured during the course of colloidal



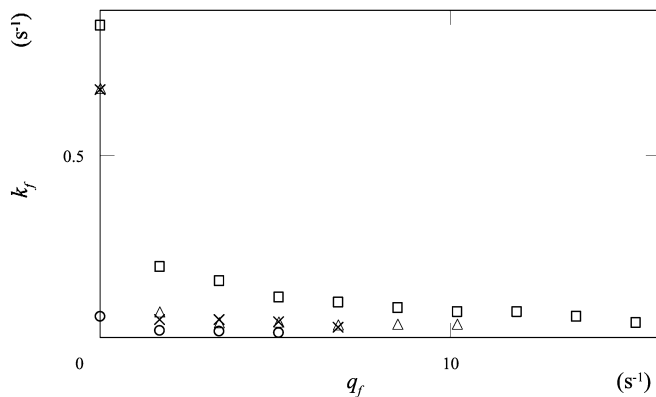
**Fig. 4** Reflection peak wavelengths ( $\lambda_p$ ) at 19.4 s after stopping the flow, as a function of initial shear rate ( $q_i$ ). CS91 spheres at 25 °C;  $q_f=0 \text{ s}^{-1}$ ;  $\phi=0.005$



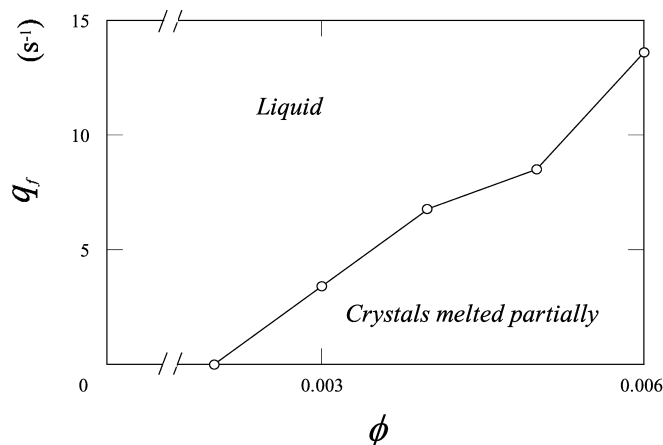
**Fig. 5** Reflection peak intensities during colloidal crystallization of CS91 spheres at 25 °C.  $\phi=0.006$ ;  $q_i=34 \text{ s}^{-1}$ ;  $q_f=3.4 \text{ s}^{-1}$

crystallization, with initial and final suspension flows at  $34 \text{ s}^{-1}$  and  $3.4 \text{ s}^{-1}$ , respectively. In this figure, the suspension was initially circulated at the shear rate of  $34 \text{ s}^{-1}$  for more than three minutes. Then the flow rate was dropped immediately to  $q_f=3.4 \text{ s}^{-1}$ . After dropping the shear rate at  $t=0$ , the intensity of the Bragg peak increased sharply. In the figure, a large number of pulses can be observed. The frequency of the pulses coincided with that of mechanical rotation of the peristaltic pump. Therefore, the remaining pulsating flow of the suspension may cause the crystal growth and melting process periodically.

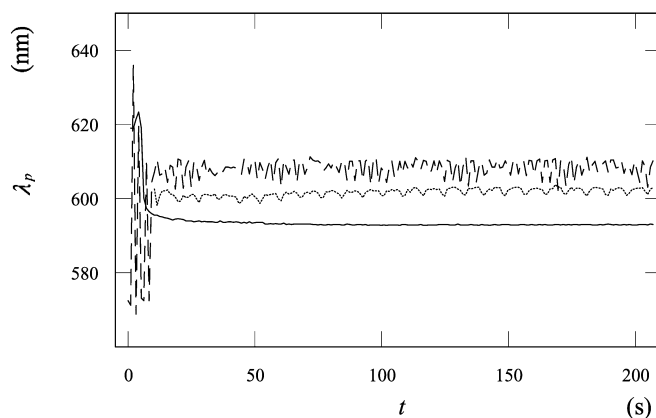
Figure 6 shows the rate coefficient  $k_f$  as a function of  $q_f$  for a sphere concentration of 0.003 to 0.006 (volume fraction). Measurements were also made for  $\phi=0.002$  suspension in this work. However, no crystals were formed. Crystallization did not occur at high  $q_f$  values either; for example at  $> 5.1 \text{ s}^{-1}$  for  $\phi=0.003$ ,  $> 6.8 \text{ s}^{-1}$  for  $\phi=0.004$  and  $> 10.2 \text{ s}^{-1}$  for  $\phi=0.005$ . At  $\phi=0.003$ , crystals were formed, but the Bragg peaks were very faint.



**Fig. 6** Rate coefficients ( $k_f$ ) as a function of final shear rate ( $q_f$ ), CS91 spheres at 25 °C;  $q_i = 34 \text{ s}^{-1}$ ; unfilled circles:  $\phi = 0.002$ ;  $\times$ :  $\phi = 0.003$ ; triangles:  $\phi = 0.004$ ; squares:  $\phi = 0.005$ ; filled circles:  $\phi = 0.006$



**Fig. 8** Relationship between final shear rate ( $q_f$ ) and volume fraction ( $\phi$ ) for the colloidal recrystallization of CS91 spheres at 25 °C.  $q_i = 34 \text{ s}^{-1}$



**Fig. 7** Time dependence of the reflection peak wavelength during the colloidal crystallization of CS91 spheres at 25 °C.  $\phi = 0.006$ ;  $q_i = 34 \text{ s}^{-1}$ ;  $q_f = 0 \text{ s}^{-1}$  (solid curve);  $q_f = 3.4 \text{ s}^{-1}$  (dotted curve);  $q_f = 6.3 \text{ s}^{-1}$  (broken curve)

Figure 7 shows the time dependence of the reflection peak wavelengths ( $\lambda_p$ ) for colloidal crystallization at  $q_f = 0 \text{ s}^{-1}$ ,  $3.4 \text{ s}^{-1}$  and  $6.3 \text{ s}^{-1}$ . Clearly  $\lambda_p$  increased as  $q_f$  increased, which supports the idea that the crystals formed in the shear flow are less dense compared to those formed at  $q_f = 0$ .

Figure 8 shows the relationship between  $q_f$  and  $\phi$  for colloidal crystallization. Most crystals coexisted with the melted liquid. Stable crystals were formed only when  $\phi$  was larger than 0.004 and  $q_f$  was smaller than  $3.4 \text{ s}^{-1}$ .

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